Dynamic portfolio Value at Risk estimation using DCC-MGARCH model

Kiumars Shahbazi & Ahmad Molabahrami

Abstract:

This paper empirically estimated the dynamic value at risk of an international portfolio consist of five ASEAN countries stock markets and time varying beta index of these stock markets by using dynamic conditional correlation GARCH models (DCC) especially during the recent 2008-2009 global financial crisis. We apply the weekly stock returns data for these 5 international stock markets from January 2005 to November 2012. The results show that, there is no evidence of global financial crisis contagion to stock markets of ASEAN countries based on time varying beta indices of these stock markets. In the other hand, during the global financial crisis period, there is significant reduction in dynamic portfolio value at risk level, that it is an evidence of lack of financial crisis contagion to these stock markets. This also means that the ASEAN portfolio leads to a low level of capital required and also a small revision to required capital. The estimated results also show that there are significant interaction and co movements in cross ASEAN stock markets except Malaysia stock market. This means that there are not diversification gains of the constructed portfolio among these stock markets. These results may have implication for investors and portfolio managers in order to international portfolio diversification and risk management.

Keyword: Time varying beta, dynamic value at risk, financial crisis contagion, portfolio, DCC-MGARCH model.

1. Introduction

In risk management literature the Value at Risk of a given constructed portfolio is one of the most important measures that is required by financial institutions for capital allocation and portfolio management worldwide. Value at Risk is a procedure designed to forecast

1 Corresponding Author, Faculty Member of Department of Economics, Urmia University, Iran, Email: k.shahbazi@urmia.ac.ir.
the worst expected loss over a given time interval and at a given confidence level (Jorion, 1996). This measure can be specified according to systematic risk or beta index of any assets in portfolio and variance of portfolio. Based on CAPM theory of financial assets (Sharpe, 1965 and Lintner, 1965), in the international investment and risk management, beta index shows the influence of a stock market from global stock index. This index is calculated by dividing covariance of market stock returns and global returns index on variance of global returns index. In the other hand, unlike to CAPM theory, it is empirically considered that the beta of stock returns and thus portfolio is time varying (Fabozzi & Francis, 1978). The econometric literatures of financial variable have indicated that the variance of a portfolio consisting of volatile assets must be time variant (Brock, 2008). Thus we can expect that Value at Risk of An internationally diversified portfolio must be time variant. Now to estimate the dynamic Value at Risk of a portfolio, the time varying beta index of individual stocks contained in portfolio and time varying variance-covariance matrix in cross stocks must be estimated.

In financial econometrics, univariate and multivariate GARCH (MGARCH\(^2\)) models have been widely used in order to estimate the time varying variance-covariance matrix of some financial assets or some international stock markets (Brock, 2008). One of the most flexible recent versions of MGARCH models is Dynamic Conditional Correlation (DCC) model which is introduced by Engle and Sheppard (2001) and Engle (2002) and is widely used to investigate dynamic relationship and contagion effects in recent studies (Kazi and et.al, 2011; Kim and Kim, 2011; Manolis, and Georgios, 2011; Min and Hwang, 2012; Guesmi and et.al, 2013; Wang and Lai, 2014). Some studies used DCC and other MGARCH models to estimate Value at Risk of financial variables (Lee and et.al, 2006; Pesaran and Pesaran, 2010; Nyssanov, 2013; Francq and Zakoian, 2014).

The recent global financial crisis in 2008-2009 periods influenced widely many international stock markets. One of the most important issues of financial crises transmission is that gains from international diversification decline. It is clear that higher correlation accomplished by higher volatility that may disrupt portfolio choice decisions.

\(^2\) Bollerslov and et al VECH model (1988), Bollerslov CCC mode (1990), Engle & Kroner BEKK model (1994) and also Engle & Sheppard (2001) and Engle DCC model (2002) are the most famous theatrical econometrics studies that are excites in MGARCH literature.
Also during the financial contagion period, the ability of individual financial institutions decreases against foreign risks. The stock markets of ASEAN region collected from a set of emerging stock markets that can be appealing to international investors to invest and have diversification opportunities. Hence we empirically investigate risk management and gains of diversifications of ASEAN stock markets through dynamic correlation, time varying beta and especially dynamic Value at Risk of portfolio consist of these stock markets especially during the recent global financial crisis.

Hence the main goal of this paper is to study empirically dynamic Value at Risk of a portfolio consisting of ASEAN stock markets and analysis the contagion recent 2008-2009 global financial crisis and diversification gains of international constructed portfolio of ASEAN stock markets based on DCC model that it is our main contribution in this paper. The novel achievements of this paper can be summarized as follows.

There is no evidence of global financial crisis contagion to stock markets of ASEAN countries based on time varying beta indexes of these stock markets. Based on DCC model estimation results, there are significant interaction and co movements in cross ASEAN stock markets except Malaysia stock market. This means that there are not diversification gains of the constructed portfolio among these stock markets. Also there is significant reduced in dynamic portfolio Value at Risk level during 2008-2009, that it is an evidence of lack of financial crisis contagion to these stock markets. Hence the ASEAN portfolio leads to a low level of capital required and also a small revision to required capital.

The rest of this paper is organized as follow. In section 2 we specify the econometrics methodology. In section 3 the data used in this study is specified. Section 4 presents the empirical results on time varying beta, interaction and dynamic relationship among ASEAN stock markets and dynamic international portfolio Value at Risk estimation of ASEAN stock markets and comparison it’s time trend in both two before 2008-2009 and after this periods. Section 5 concludes the paper with summary.

2. Methodology
In this section, we first specify the dynamic portfolio value at risk formula based on dynamic variance-covariance matrix among portfolio assets and time varying beta of
individual stock markets. Then we introduced to econometrics literature of MGARCH models in order to estimate dynamic variance-covariance matrix of financial time series.

a. Portfolio Dynamic Value at Risk Calculation

In this section we specify dynamic portfolio value at risk formula based on time varying beta and variance and covariance matrix among of portfolio stocks returns. Suppose we have an international portfolio consisting P stock markets and $\Sigma$ is the variance–covariance matrix among these stock markets. However the weight vector of these stock markets is $w$ where $w' = [w_1 \ldots w_P]_{1 \times P}$. The value at risk of this portfolio can be evaluated as:

$$VaR_t = z \sqrt{w' \beta' \Sigma_w^2} \sqrt{\Delta t}$$

(1)

Where $\sigma_p^2 = w' \beta' \Sigma_w \beta w$ is the variance of portfolio, and $z$ display the critical standard normal distribution in a given confidence level. The time interval $\Delta t$ is the time horizon.

In the other hand if $\beta$ be the systematic risk of portfolio, we can write:

$$\beta = \sum_{i=1}^{P} w_i \beta_i + w_2 \beta_2 + \ldots + w_P \beta_P$$

(2)

In equation (2), $\beta_i$ is the systematic risk of i-th stock market in the international portfolio, where it is calculated in equilibrium based on CAPM theory as:

$$\beta_i = \frac{Cov(R_i, R_m)}{Var(R_m)}$$

(3)

In equation (3), $R_i$ and $R_m$ are the i-th market returns and global returns respectively.

The MGARCH models give an ability to have variance of any stocks and covariance among any two stocks in portfolio at time t. Thus we can calculate systematic risk of any stocks and so we can specify Value at Risk of the portfolio in any time t. thus the dynamic calculation formulas of the Value at Risk of a portfolio in any confidence level and specified time horizon can be written as:
\[
\beta_t = w_{1,t} \beta_{1,t} + w_{2,t} \beta_{2,t} + \ldots + w_{p,t} \beta_{p,t}
\]  
(4)

\[
\sigma^2_{p,t} = \sum_{i=1}^{n} w_{i,t}^2 \sigma^2_{i,t} + \sum_{i=1}^{n} \sum_{j=1}^{n} 2w_{i,t}w_{j,t} \rho_{ij,t} \sigma_{i,t} \sigma_{j,t}
\]  
(5)

\[
VaR_{p,t} = z \sigma_{p,t} \sqrt{\Delta t} W_t
\]  
(6)

Where $\sigma^2_{i,t}$ is the variance of the $i$-th market returns in time $t$. Also $\rho_{ij,t}$ is the correlation between $i$-th and $j$-th markets returns in time $t$. In the other hand $\beta_t$ is the international portfolio systematic risk in time $t$ that can be expressed as weighted average of individual markets beta.

Hence if the international time varying variance covariance matrix among markets returns and global returns be specified, we can estimate Value at Risk of international portfolio in any time moment in study period.

b. MGARCH models

A general class of multivariate conditional volatility and correlation models is the MGARCH models specification (Bollerslov and et.al, 1988, Engle and Kroner, 1995). One of the most important challenges with MGARCH models is that the number of unknown parameters rises unbounded with the dimension of model. The Diagonal-VECH and BEKK version of MGARCH models is more parsimonious in number of unknown parameters but still contain many parameters that must be estimated.

The Dynamic Conditional Correlation (DCC) model that introduced by Engle and Sheppard (2001) and Engle (2002) which it is generalized of Constant Conditional Correlation (CCC) model of Bollerslov (1990) by allowing for time variation in correlation matrix and modeled in term of a small number of unknown parameters is one of the most flexible versions of MGARCH models.

Consider residual matrix obtained from univariate AR estimated models of returns time series and is the information matrix in time $t-1$, and then DCC-MGARCH model is specified as:
\[ x_t | \Phi_{t-1} \approx N(0, D_t R_t D_t) \]  
\[ D^2_t = \text{diag} (\omega_t) + \text{diag} (\kappa_t) x_{t-1} x_{t-1}' + \text{diag} (\lambda_t) D^2_{t-1} \]  
\[ \varepsilon_t = D_t^{-1} x_t \]  
\[ Q_t = S(11' - A - B) + A \varepsilon_{t-1} \varepsilon_{t-1}' + BQ_{t-1} \]  
\[ R_t = \text{diag}(Q_t)^{-1} Q_t \text{diag}(Q_t)^{-1} \]  

The bivariate specification of equation (10) is:

\[ Q_t = S(1-\alpha - \beta) + \alpha \varepsilon_{t-1} \varepsilon_{t-1}' + \beta Q_{t-1} \]  

Where in equation (7) it is assumed that residuals normally distributed with mean 0 and variance as: \( D_t R_t D_t \) and \( D_t \) is the diagonal variance matrix and \( R_t \) is the time varying correlation matrix among residuals. In equation (8), diagonal variance matrix specified as a GARCH (1, 1) process. Equations (10) and (12) are key equations of DCC model. The \( Q_t \) is variance covariance matrix in time t and A and B are matrix of unknown parameters of DCC model. In bivariate specification, \( \alpha \) and \( \beta \) are unknown parameters that indicated the reaction of conditional volatility to markets and global shock and persistence in conditional volatility irrespective of anything happening in the market respectively.

From the equation (10) and (11) we can obtain time varying variance covariance and correlation matrixes that required for estimating time varying beta index and Value at Risk of portfolio based on equations (3) and (6) respectively.

3. Data

The study period extends from January 2005 to November 2012. This period is long enough to include the recent global financial crisis in 2008-2009 periods. We use the weekly famous equity index dataset of ASEAN countries includes Indonesia (Jakarta Comp index), Malaysia (KLSE Comp index), Philippine (Philad Comp index), Singapore (Straits Times index), Thailand (SET index) and Global S&P 1200 index as world index. For all stock price series, weekly returns are defined as:

\[ r_{p_t} = 100 \log(p_t / p_{t-1}) \]
Where $p_t$ and $p_{t-1}$ indicated stock price in t and t-1 period respectively (Asteriou, D. 2006). Table 1 presents summary statistics and normality test. No one of time series distributed normally Based on Jarque-Bera statistics.

**Table 1.** Summary statistics for weekly data returns

<table>
<thead>
<tr>
<th>Equities</th>
<th>Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>KLSE Comp index</td>
<td>0.15</td>
</tr>
<tr>
<td>Jakarta Comp index</td>
<td>0.35</td>
</tr>
<tr>
<td>Straits Times index</td>
<td>0.095</td>
</tr>
<tr>
<td>Philad Comp index</td>
<td>0.26</td>
</tr>
<tr>
<td>SET index</td>
<td>0.16</td>
</tr>
<tr>
<td>World index</td>
<td>0.03</td>
</tr>
</tbody>
</table>

**Source:** researcher's calculation

The following figures display the time series of indices returns time series over under study period.
In the next section the empirical results are presented.

4. Empirical results
In this section, first univariate GARCH of every 6 variables is estimated. These results are displayed in table 2, where $\kappa_i$ and $\lambda_i$ are univariate ARCH and GARCH coefficients.
Table 2. Maximum likelihood estimation of the univariate GARCH (1, 1) parameters

<table>
<thead>
<tr>
<th>Returns series</th>
<th>GARCH parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \omega )</td>
</tr>
<tr>
<td>KLSE Comp index</td>
<td>0.13(0.00)</td>
</tr>
<tr>
<td>Jakarta Comp index</td>
<td>1.45(0.00)</td>
</tr>
<tr>
<td>Straits Times index</td>
<td>0.3 (0.00)</td>
</tr>
<tr>
<td>Philad Comp index</td>
<td>0.93(0.00)</td>
</tr>
<tr>
<td>SET index</td>
<td>1.07(0.00)</td>
</tr>
<tr>
<td>World index</td>
<td>0.68(0.00)</td>
</tr>
</tbody>
</table>

Source: researcher's calculation

The univariate GARCH estimation results show that the Jakarta stock market has maximum ARCH coefficient and minimum GARCH coefficient among five under studied stock markets, which it means that Jakarta stock market returns volatility is very sensitive to market events but not permanently. In the next step of empirical analysis, the DCC-GARCH model is estimated between world index returns and any 5 ASEAN countries stock market returns. This result is presented in table 3.

Table 3. Maximum likelihood estimation of the DCC-GARCH (1, 1) parameters between global returns and individual ASEAN stock markets returns

<table>
<thead>
<tr>
<th>Returns series</th>
<th>DCC parameters</th>
<th>LogL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \alpha )</td>
<td>( \beta )</td>
</tr>
<tr>
<td>KLSE Comp index</td>
<td>-0.00(0.87)</td>
<td>0.44(0.93)</td>
</tr>
<tr>
<td>Jakarta Comp index</td>
<td>-0.00(0.61)</td>
<td>-0.8(0.01)</td>
</tr>
<tr>
<td>Straits Times index</td>
<td>-0.00(0.27)</td>
<td>1.00(0.00)</td>
</tr>
<tr>
<td>Philad Comp index</td>
<td>-0.00(0.74)</td>
<td>0.82(0.29)</td>
</tr>
<tr>
<td>SET index</td>
<td>-0.01(0.00)</td>
<td>1.005(0.00)</td>
</tr>
</tbody>
</table>

Source: researcher's calculation
In the table 3, the estimated DCC coefficients $\alpha$ and $\beta$ are displayed based on equation (12). The $\alpha$ coefficient measures the reaction of conditional volatility to markets and global shock and the $\beta$ coefficient measures the persistence in conditional volatility irrespective of anything happening in the market. The estimation results that are presented in table 3 show that, Thailand stock market has maximum $\alpha$ and $\beta$ DCC coefficients among other under study stock markets. In the other hand Malaysia stock market has the minimum DCC coefficients. These results indicated that Thailand and Malaysia stock markets have maximum and minimum effects and sensitivity from global variations among other ASEAN countries.

From the DCC model, we estimate the time varying variance-covariance matrix and correlation among any countries stock market returns that mentioned. Now we are able to calculate time varying beta index of any 5 stock markets based on equation (3).

In figure 2 the time varying Beta index of 5 stock markets have been shown. It is clear that every 6 stock markets have low relationship with global trend of financial markets. During the 2008-2009 financial crisis periods the time varying beta index of Thailand and Singapore stock markets significantly decline, that it is evidences of low correlation and transmission and contagion effects of financial crisis to these stock markets. Thus, there are some diversifications gains of international portfolio consist of ASEAN countries.
Fig 2. Time varying stock markets systematic risk obtained from DCC-MR model.
In the following table the estimation results of DCC-GARCH among 5 international stock markets that contained in our constructed portfolio, are presented:

Table 4. Maximum likelihood estimation of the DCC-GARCH (1, 1) Parameters among ASEAN stock returns

<table>
<thead>
<tr>
<th>Returns series</th>
<th>DCC parameters</th>
<th>LogL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\alpha)</td>
<td>(\beta)</td>
</tr>
<tr>
<td>Jakarta Comp index &amp; KLCSE Comp index</td>
<td>-0.06(0.04)</td>
<td>0.84(0.00)</td>
</tr>
<tr>
<td>Jakarta Comp index &amp; SET index</td>
<td>-0.03(0.33)</td>
<td>-0.99(0.00)</td>
</tr>
<tr>
<td>Jakarta Comp index &amp; Straits Times index</td>
<td>0.06(0.04)</td>
<td>0.87(0.00)</td>
</tr>
<tr>
<td>Jakarta Comp index &amp; Philad Comp index</td>
<td>0.03(0.04)</td>
<td>0.85(0.00)</td>
</tr>
<tr>
<td>KLCSE Comp index &amp; SET index</td>
<td>-0.01(0.71)</td>
<td>0.66(0.65)</td>
</tr>
<tr>
<td>KLCSE Comp index &amp; Straits Times index</td>
<td>0.08(0.03)</td>
<td>0.86(0.00)</td>
</tr>
<tr>
<td>KLCSE Comp index &amp; Philad Comp index</td>
<td>0.05(0.27)</td>
<td>0.55(0.32)</td>
</tr>
<tr>
<td>SET index &amp; Straits Times index</td>
<td>0.15(0.00)</td>
<td>-0.103(0.05)</td>
</tr>
<tr>
<td>SET index &amp; Philad Comp index</td>
<td>-0.017(0.32)</td>
<td>0.87(0.00)</td>
</tr>
<tr>
<td>Straits Times index &amp; Philad Comp index</td>
<td>0.04(0.04)</td>
<td>0.85(0.00)</td>
</tr>
</tbody>
</table>

Source: researchers’ calculation

It is interesting that there is no significant interaction between Malaysia and Thailand stock markets. Also there is no significant interaction between Malaysia and Philippine stock markets. But other stock markets have significant high level of interaction and are related together. In whole, there are high significant interactions and co-movements in cross ASEAN stock markets except Malaysia stock market. Thus there is not any evidence of diversification benefits of ASEAN stock markets.

Now we can estimate the international portfolio dynamic value at risk by using time varying variance-covariance matrix obtained from estimated DCC-GARCH. In the figure 3 the time trend of dynamic portfolio value at risk is displayed. It is assumed that international portfolio consist of ASEAN stock markets with same weights. Our results show falling dynamic portfolio Value at Risk during 2008-2009 period. It seems that the
global financial crisis in 2008-2009 periods had no effects on ASEAN region stock markets.

As can be seen in figure, the dynamic international ASEAN portfolio value at risk at 90, 95 and 99 percentile confidence levels is time variant. Furthermore, it is interesting to observe that averages Value at Risk at different confidence level are: 0.4, 0.48 and 0.62 percent with standard deviation of 0.06, 0.07 and 0.1, respectively. This means that the ASEAN portfolio lead to a low level of capital required and also a small revision to required capital. It is interesting that during the global financial crisis, there is a significant reduction in dynamic portfolio Value at Risk level, evidence to not contagion effects of global financial crisis to these stock markets.

5. Conclusion
This paper has empirically analyzed the application of DCC-MGARCH model to estimation time varying beta, mutual interaction and volatility transition among ASEAN stock markets and estimating dynamic Value at Risk for a constructed portfolio with ASEAN countries stock markets. We estimated Value at Risk for an equally weighted portfolio consisting of these stock markets in 90, 95 and 99 percentile confidence levels. The results show that, there are not diversification gains of the constructed portfolio among these stock markets. Thus, it is recommended that international investors consider this issue in order to portfolio risk management. In the other hand, based on the main
results, there is significant reduction in dynamic portfolio Value at Risk level during 2008-2009, that is an evidence of lack of financial crisis contagion to these stock markets. Hence, the ASEAN portfolio leads to a low level of capital requirements and also a small revision to required capital and it seems that the ASEAN stock markets have a safe environment for international investors and risk managers. For future research it is suggested to compare performance of other MGRACH models such as symmetric and asymmetric BEKK and CCC models with DCC model. It is also recommended to investigate the contagion effects among international stock markets based on Regime-Switching GARCH models.

References


